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One Loop Chiral Corrections to Hard Exclusive Processes: I. Pion Case

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Abstract

We computed the leading non-analytic chiral corrections to the generalized parton distributions (GPDs) of the pion and to the two-pion distribution amplitudes. This allows us to obtain the corresponding corrections for the hard exclusive processes, such as $\gamma^*\gamma \to \pi\pi$, $\gamma^* N \to 2\pi N'$ and deeply virtual Compton scattering on the pion target.

1 Introduction

Hard processes are known to provide us with valuable information about the quark and gluon structure of hadrons in terms of parton distributions and parton distribution amplitudes. The generalized parton distributions (GPDs) [1, 2, 3, 4], entering the QCD description of the hard exclusive processes, interpolate, in a sense, between usual parton distribution, distribution amplitudes and elastic hadron form factors (for a review see e.g. [5]). GPDs are determined by the low energy physics, therefore their dependence on the quark mass, small momentum transfer, etc. can be studied with help of chiral perturbation theory (ChPT).

In the present paper we develop the ChPT for the simple case of the GPDs in the pion and two pion distribution amplitudes (2πDA). We present the results at the one loop level of the ChPT. In this way we compute the leading non-analytic corrections of the type $p^2 \ln(p^2)$ (where $p^2 \sim m_\pi^2 \sim t$) to GPDs. Such corrections are universal and allow us to get an insight into structure of the GPDs. Additionally the leading non-analytic chiral correction to GPDs can be immediately translated to corresponding correction for the exclusive hard processes such as $\gamma^*\pi \to \gamma\pi$, $\gamma^* N \to 2\pi N'$, $\gamma^*\gamma \to \pi\pi$, etc. Such chiral corrections to the hard exclusive processes are computed in the present paper for the first time.

2 Chiral expansion for the light-cone matrix elements

In this section we discuss the matching of the light-cone quark-gluon operators to the operators in the effective field theory.

The generalized parton distributions (GPDs) and distributions amplitudes are defined as various matrix elements of the quark-gluon operators on the light cone. Let us introduce left and right twist-2 quark operators on the light cone:

$$
\begin{aligned}
O^L_{fg}(\lambda) &= \bar{\psi}_g\left(\frac{\lambda n}{2}\right) \not{n}\, \frac{1+\gamma_5}{2}\, \psi_f\left(-\frac{\lambda n}{2}\right)\,, \\
O^R_{fg}(\lambda) &= \bar{\psi}_g\left(\frac{\lambda n}{2}\right) \not{n}\, \frac{1-\gamma_5}{2}\, \psi_f\left(-\frac{\lambda n}{2}\right)\,.
\end{aligned}
\tag{1}
$$

Here the vector n^μ is the light-cone vector $n^2 = 0$, f, g stand for flavour indices. It is always assumed the colour gauge link along a straight line between the points $\lambda n/2$ and $-\lambda n/2$. In the effective field theory the operators (1) are matched to the operators formulated in terms of effective degrees of freedom:

$$
O^{L,R}(\lambda) = F \otimes O^{L,R}_{\text{eff}}(\lambda)\,, \tag{2}
$$

where $O^{L,R}_{\text{eff}}(\lambda)$ is an effective hadronic operator with the same quantum numbers (but not necessarily with the same twist) as the quark operators (1) and F is the generating function for the c-number coefficients which are input for effective field theory. In order to make sense out of decomposition (2) we need to have systematic power counting rules for construction of the hadronic operator $O^{L,R}_{\text{eff}}(\lambda)$.

As usually we are going to use the Goldstone bosons of spontaneous chiral symmetry breaking as degrees of freedom for the construction of the effective operators. The standard power counting of the chiral perturbation theory (ChPT) uses the fact that the Goldstone bosons do not interact at zero momentum. Therefore on the level of this effective field theory, the expansion amounts to a derivative expansion of the effective Lagrangian [6] (see [7] for introduction to ChPT). The lowest order term reads:

$$
\mathcal{L}_{eff} = \mathcal{L}_2 + \ldots = \frac{F_\pi^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle + \ldots \tag{3}
$$

where the Goldstone boson fields $U(x) = \exp(i\pi^a(x)\tau^a/F_\pi)$, $\chi = 2B\mathrm{diag}(m_u, m_d)$[1] and dots denote higher order terms. We consider two flavour case and do not introduce external fields for brevity. The fields U, χ that occur in the effective Lagrangian are subject to the following chiral counting rules:

$$
U \sim O(p^0), \quad \partial_\mu U \sim O(p^1), \quad \chi \sim O(p^2) \tag{4}
$$

where p is small momentum, i.e. small parameter of chiral expansion. From (4) one finds that the leading order effective Lagrangian (3) is of order p^2. We would like to emphasize, that locality together with condition $UU^\dagger = 1$ plays important role in derivation of the chiral expansion [6].

On the other hand, the description of the many hadron hard reactions grounds on the QCD collinear factorization. In such approach the non-perturbative part associated with a soft physics is parametrized by the matrix elements of some non-local light-cone operators. These objects appear as natural constructing blocks and it is convenient to

[1] We use standard notation for quark condensate $\langle\bar{\psi}\psi\rangle = -F_\pi^2 B + O(p^2\ \ln p)$

keep these operators non-local without transition to a series of the local operators. Such approach is useful in the higher energy phenomenology and we would like to follow this philosophy in the effective theory.

Therefore, to perform the matching (2) in terms of the effective fields the standard counting rules (4) should be slightly extended. The point is that although after the QCD factorization the soft part of the hard processes does not contain the hard momenta, it still "remembers" about them. In the operators (1) such "memory" is reflected by the dependence on the light-cone vector n^μ and we have to specify the chiral order of this parameter.

Let us mention that the light-cone decomposition of any four-vector V^μ reads:

$$V^\mu = V^+ \, \tilde{n}^\mu + V^- \, n^\mu + V_\perp^\mu \, . \tag{5}$$

Here are n^μ and $\tilde{n}^\mu$ are light-cone vectors $n^2 = \tilde{n}^2 = 0$ which we normalize as $n \cdot \tilde{n} = 1$. These two vectors define two-dimensional plane, the perpendicular plane is called transverse plane. The vectors from the transverse plane $V_\perp^\mu$ by definition satisfy $n \cdot V_\perp = \tilde{n} \cdot V_\perp = 0$. The physical observables are obviously invariant under rescaling of the vector n, *i.e.* under transformation $n^\mu \to c\, n^\mu$ where c is an arbitrary nonzero constant. This invariance corresponds to the boost invariance of the physical observables. It is convenient to fix the normalization of the light-cone vector n^μ by condition like $n \cdot p = 1$ where p is one of the small external momenta entering the soft part of the amplitude. Such condition implies that the light-cone vectors $n^\mu \sim O(1/p)$ and $\tilde{n}^\mu \sim O(p)$, where p assumed to be a generic soft momentum as in power counting (4).

To summarize, we have to construct the effective hadronic operator in eq. (2) using as building blocks chiral fields $U(x)$ and their derivatives with counting rules:

$$n \cdot \partial U(x) \sim O(p^0), \quad \tilde{n} \cdot \partial U(x) \sim O(p^2), \quad \partial_\perp U(x) \sim O(p) \, . \tag{6}$$

Using these building blocks one can derive that in effective field theory the operators (1) are matched to the operators in terms of Goldstone degrees of freedom with the same quantum numbers:

$$\begin{aligned} O^L_{fg}(\lambda) &= \frac{iF_\pi^2}{4} \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \; F(\beta,\alpha) \left[U\left(\frac{\alpha+\beta}{2}\lambda n\right) n\cdot \overleftrightarrow{\partial}\, U^\dagger \left(\frac{\alpha-\beta}{2}\lambda n\right)\right]_{fg} + \ldots \, , \\ O^R_{fg}(\lambda) &= \frac{iF_\pi^2}{4} \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \; F(\beta,\alpha) \left[U^\dagger\left(\frac{\alpha+\beta}{2}\lambda n\right) n\cdot \overleftrightarrow{\partial}\, U \left(\frac{\alpha-\beta}{2}\lambda n\right)\right]_{fg} + \ldots \end{aligned} \tag{7}$$

Here $F(\beta,\alpha)$ is the generating function of the tower of low-energy constants and $\overleftrightarrow{\partial}$ denotes $\overrightarrow{\partial} - \overleftarrow{\partial}$. The low-energy constants are characteristics of the structure of the pion, they are not determined in the effective field theory. The ellipsis in eqs. (7) stands for operators which do not contribute to the one and two pion matrix elements of the operators $O^{L,R}$ or which are of higher orders in the chiral counting. Note that if one would consider the chiral corrections say for three pion distribution amplitudes one would need to add additional operators to eq. (7). In the next section we consider chiral expansion of some light-cone matrix elements at the leading order using formulae (7).